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IN-TRAIL PROCEDURE OCEANIC SAFETY ASSESSMENT

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SUMMARY

This paper presents a method to calculate the probability of longitudinal overlap during an ITP maneuver. It is assumed that longitudinal overlap at co-altitude will lead to a mid-air collision.

1 Introduction

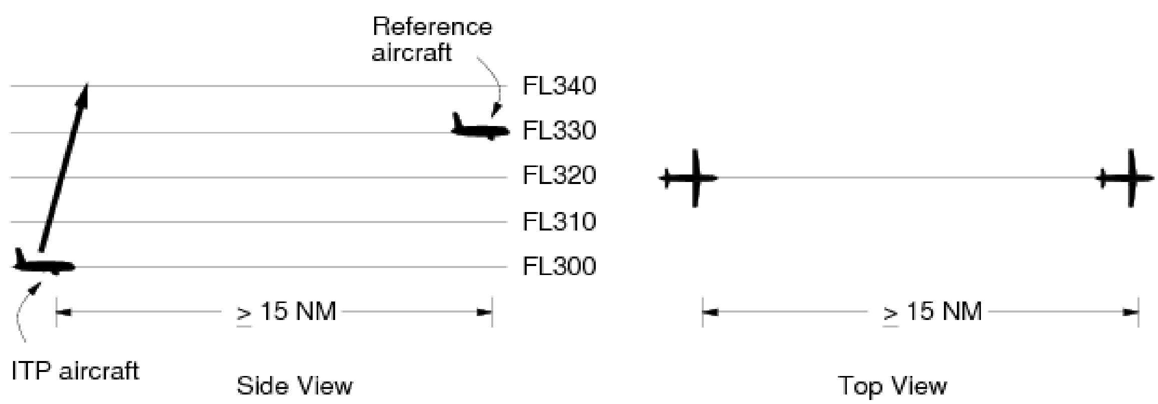
The In-Trail Procedure (ITP) uses GNSS/GPS/ADS-B data to apply distance based longitudinal separation. The ITP Operational Description was presented at the 10th SASP – WG/WHL meeting and is summarized in SASP-WG/WHL/10-WP/21. At this meeting, a preliminary safety and collision study [1] was presented to the Mathematicians Sub Group (MSG). It was agreed by the MSG that the safety and collision study was going to be redone using a method and format that more closely follows previous safety studies performed by members of the MSG. The collision risk assessment and collision risk model was presented at the 11th SASP-WG/WHL and several recommendation were made.

This Working Paper is the collision risk safety assessment of the In-Trail Procedure and follows the recommendations given at the 10th and 11th SASP meeting and subsequent recommendations. It calculates the probability of aircraft longitudinal overlap based on given values of accuracy for GNSS/ADS-B, altitude error, latency error, initiation criteria parameters for the ITP, and a wind model. A parametric analysis was performed in [1] to determined the sensitivity of collision risk to accuracy, integrity and initiation criteria. Although the In-Trail Procedure is defined for aircraft along the same track, as defined in Doc. 4444 - PAN ATM [2], this working paper calculates the probability of longitudinal overlap when the angle between the aircraft tracks is zero. That is, identical same tracks.

The probability is calculated for a pair of aircraft along the zero angle identical same tracks, in the same direction and with one aircraft climbing or descending and the other aircraft not maneuvering. Both aircraft have GNSS/GPS equipment on board.

The aircraft climbing or descending will be the aircraft cleared to perform the ITP maneuver and will be called the ITP aircraft. The aircraft not maneuvering will be called the Reference aircraft. The Reference aircraft has ADS-B equipment which transmits its identification, position, velocity, altitude, and other information. The ITP aircraft has ADS-B equipment that transmits and *receives* ADS-B information. Thus, the ITP aircraft has the state of the Reference aircraft and its own state information. It is based on this information that the ITP request is made. The ITP aircraft request for an ITP procedure includes the distance between the two aircraft. A detailed description of the procedure can be found in reference [5]. Figure 1.1 shows an illustration of a side and top view of an ITP maneuver example.

Figure 1.1. Side and Top View of ITP Maneuver Example.



The ITP establishes that an aircraft performing an ITP maneuver must be a minimum distance of 15 nautical miles in front of or behind a Reference aircraft before requesting and initiating an ITP climb or descent. This distance, in conjunction with other requirements, is the initiation criteria standard for ITP. The reported distance *X*, $X \geq 15\ NM$, is the distance between the ITP aircraft and the Reference aircraft as measured by the ITP aircraft. The reported distance *X* is based on the ITP GNSS data and the Reference aircraft GNSS data as reported by an ADS-B link.

The probability of longitudinal overlap given an initial distance "*X*" is calculated for the following errors and affecting sources:

- Position error
- Velocity error
- Altitude error
- Latency error
- Winds

The altitude error component does not directly affect the probability of longitudinal overlap. It has an indirect effect in a climb/descent by increasing or decreasing the time which takes the maneuvering aircraft (ITP aircraft) to reach the Reference aircraft's altitude.

2 Modeling the Error Terms

2.1 Position error

The true longitudinal ground location along the track of the ITP and Reference aircraft, at the time of the ITP initiation, are represented by x_{ITP} and x_{Ref}, respectively. It is assumed that the measured longitudinal ground location $\hat{x}_{ITP}$ is the true longitudinal ground location x_{ITP} plus a zero mean random error $\eta_{p\,ITP}$ with a fixed standard deviation $\sigma_{p\,ITP}$.

$$\hat{x}_{ITP} = x_{ITP} + \eta_{pITP} \tag{2.1}$$

It is also assumed that the measured longitudinal location $\hat{x}_{Ref}$ of the Reference aircraft is the true longitudinal location x_{Ref} plus a zero mean random error $\eta_{p\,Ref}$ with a fix standard deviation $\sigma_{p\,Ref}$.

$$\hat{x}_{Ref} = x_{Ref} + \eta_{p\,Ref} \tag{2.2}$$

The true longitudinal ground distance between the ITP and reference aircraft is given by:

$$x_{Ref} - x_{ITP} = \hat{x}_{Ref} - \hat{x}_{ITP} - \eta_{p\,Ref} + \eta_{p\,ITP} \tag{2.3}$$

Where $\hat{x}_{Ref} - \hat{x}_{ITP} = X$, the initial ITP distance as measured by the ITP aircraft, and $X \geq 15$ NM, the minimum distance initiation criterion.

2.2 Velocity error

In addition to the distance criterion, an aircraft requesting and performing an ITP maneuver must comply with other criteria. The criteria regarding velocity are as follow:

- The ITP aircraft must not have greater than 20 knots closing ground speed on the Reference aircraft.

- The ITP aircraft must not have greater than 0.04 Mach closing air speed on the Reference aircraft.

These two criteria are related by the wind speed at their flight levels. Reference [1] shows two dimensional and three dimensional plots which represent the interrelation of the wind, ground speed criterion and Mach criterion.

Two errors are considered for the along track speed of the aircraft.

1. Ground speed error from the GNSS for the ITP aircraft and the GNSS as reported by ADS-B for the Reference aircraft. This measurement is used for the initiation criteria.
2. Static air speed error. The difference between the commanded Mach number and the true Mach number of the aircraft.

The ground speed error and the static air speed (Mach) error are not statistically independent. Appendix A shows examples of how the error of one measurement can be affected by the other as a function of wind.

Let the true ground speeds of the ITP and Reference aircraft be represented by vg_{ITP} and vg_{Ref}, respectively. The measured ITP and Reference aircraft ground speed are the true ground speed plus an error term,

$$\hat{vg}_{ITP} = vg_{ITP} + \eta_{vg\,ITP} \tag{2.4}$$

$$\hat{vg}_{Ref} = vg_{Ref} + \eta_{vg\,Ref} \tag{2.5}$$

The ground speed initiation criterion for ITP limits the closure ground speed to 20 knots at the initiation of the maneuver. The initial conditions are denoted with the superscript *ic*,

$$\hat{vg}^{ic}_{ITP} - \hat{vg}^{ic}_{Ref} \leq 20 \text{ knots} \tag{2.6}$$

$$\hat{vg}^{ic}_{Ref} - \hat{vg}^{ic}_{ITP} \leq 20 \text{ knots} \tag{2.7}$$

Equation 2.6 applies when the ITP aircraft is the trailing aircraft and Equation 2.7 applies when the ITP aircraft is the leading aircraft. Without loss of generality, in the following derivations, we consider the case when the ITP aircraft is the trailing aircraft. The case when the ITP aircraft is the leading aircraft will follow a similar derivation.

Replacing equations 2.4 and 2.5 into 2.6 gives,

$$vg^{ic}_{ITP} - vg^{ic}_{Ref} \leq 20 - \eta_{vg\,ITP} + \eta_{vg\,Ref} \text{ knots} \tag{2.8}$$

Therefore, the true relative ground speed of the aircraft at the initiation is bounded by 20 knots plus the GNSS/ADS-B error.

The true Mach numbers for the ITP and Reference aircraft are represented by vm_{ITP} and vm_{Ref}, respectively. The measured ITP and Reference aircraft Mach numbers are the true Mach numbers plus the error:

$$\hat{vm}_{ITP} = vm_{ITP} + \eta_{vm\,ITP} \tag{2.9}$$

$$\hat{vm}_{Ref} = vm_{Ref} + \eta_{vm\,Ref} \tag{2.10}$$

The Mach initiation criterion for ITP limits the Mach difference to a closure of 0.04,

$$\hat{vm}_{ITP} - \hat{vm}_{Ref} \leq 0.04 \text{ Mach} \tag{2.11}$$

$$\hat{vm}_{Ref} - \hat{vm}_{ITP} \leq 0.04 \text{ Mach} \tag{2.12}$$

Equation 2.11 is the criterion when the ITP aircraft is the trailing aircraft and Equation 2.12 when the ITP aircraft is the leading aircraft. As with the ground speed, we consider the ITP aircraft trailing case. Replacing equations 2.9 and 2.10 in to 2.11 gives,

$$vm_{ITP} - vm_{Ref} \leq 0.04 - \eta_{vm\,ITP} + \eta_{vm\,Ref} \tag{2.13}$$

2.3 Altitude error

Altitude error affects the longitudinal distance between the aircraft by increasing or decreasing the time that the ITP aircraft requires to climb or descend through the Reference aircraft flight level. The altitude errors of the ITP and Reference aircraft are represented by $\eta_{alt\,ITP}$ and $\eta_{alt\,Ref}$ respectively.

To simplify the analysis, a conservative approach is taken to always add an additional climb or descent time t_a in the calculation of distance between the aircraft. The additional time to climb is the altitude error divided by the minimum climb/descent rate,

$$t_a = \frac{\eta_{alt\,ITP} + \eta_{alt\,Ref}}{300\,feet/min} \tag{2.14}$$

2.4 Latency error

Latency error is assumed to affect the position reporting of the aircraft. For the ITP procedure, the Reference aircraft transmits its state data to the ITP aircraft. If the Reference aircraft data is older than the ITP data, an error in the relative distance could result. There are two cases for the ITP maneuver; when the ITP aircraft is in trail and when the ITP aircraft is the leading aircraft. If the ITP is the trailing aircraft, then the latency error will actually increase the actual distance over the measured distance, as shown in Figure 2.1. If the ITP aircraft is the leading aircraft, the latency error will reduce the actual distance over the measure distance as shown in Figure 2.2.

Figure 2.1. Error latency for ITP in Trail.

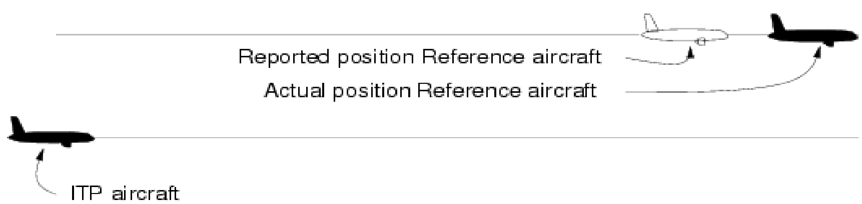
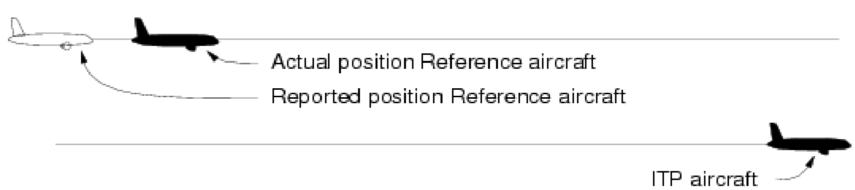


Figure 2.2. Error latency for ITP aircraft in the lead.



The worst case is assumed where the latency error always reduces the distance between the aircraft. For this risk analysis, the latency error is considered to be constant. The latency error $\eta_{latency}$ is multiplied by the speed of the aircraft to obtain the reduction in distance. The speed of the aircraft is assumed to be Mach 0.85 times the speed of sound at 35 thousand feet and International Standard Atmosphere (ISA), 576.6 knots,

$$x_{latency} = \eta_{latency} 0.85(576.6) \text{ NM} \tag{2.15}$$

3 Wind

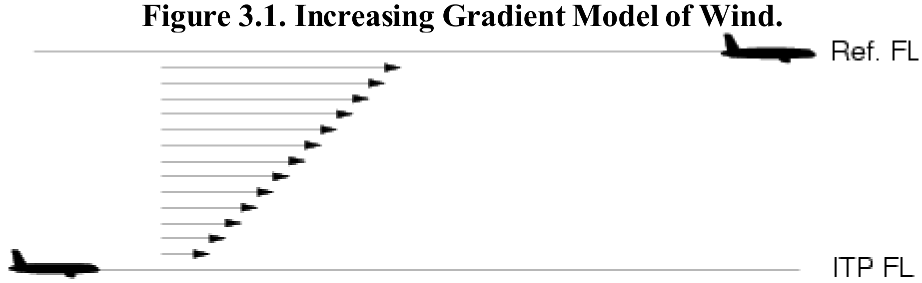
Four models of wind are considered for the collision risk analysis:

- A linearly increasing wind gradient from the ITP aircraft flight level to the Reference aircraft flight level.
- A linearly decreasing wind gradient from the ITP aircraft flight level to the Reference aircraft flight level.
- A constant wind gradient.
- An increasing and then decreasing wind gradient from the ITP aircraft flight level to the Reference aircraft flight level.

3.1 First Wind Model

The fist wind model is a linearly positive wind gradient as shown in Figure 3.1 The length of the arrows represent the magnitude of the wind at the given level.

Figure 3.1. Increasing Gradient Model of Wind.



Using this model of wind, the ITP initiation criteria bound the closure rate of the ITP aircraft on the Reference aircraft. The wind, Mach speed and ground speed are related by,

$$W + 576.6\,vm = vg \tag{3.1}$$

where W is wind, 576.6 is the speed of sound at 35 thousand feet and International Standard Atmosphere (ISA), vm is the Mach speed, and vg the ground speed. The ground speeds of the ITP and Reference aircraft are then given by

$$vg_{ITP} = 576.6\,vm_{ITP} + W \tag{3.2}$$

$$vg_{ref} = 576.6\,vm_{Ref} + W_{Ref} \tag{3.3}$$

where W is the wind speed at any level between the ITP and Reference aircraft flight levels and W_{Ref} is the wind speed at the Reference aircraft flight level. The ITP Mach criterion bounds the Mach closure to 0.04 as shown in Equation 2.13. Substituting equations 3.2 and 3.3 into Equation 2.13 we have,

$$vg_{ITP} - vg_{Ref} \leq 576.6(0.04 - \eta_{vm\,ITP} + \eta_{vm\,Ref}) + (W - W_{Ref}) \quad (3.4)$$

Because the wind model for the first case assumes increasing winds from the ITP aircraft flight level to the Reference aircraft flight level,

$$W_{Ref} \geq W \quad (3.5)$$

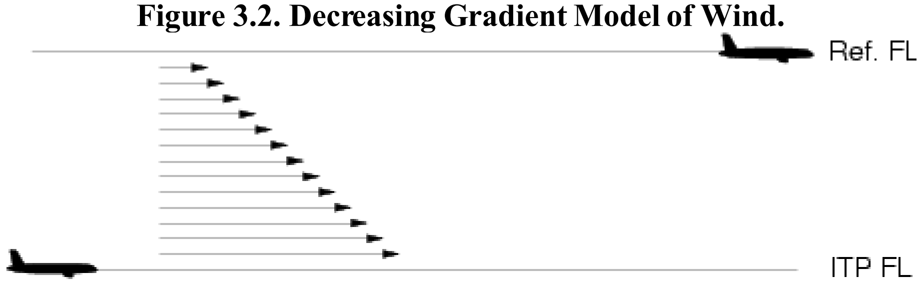
Therefore, the closure rate through the climb is always equal or less than the Mach difference criterion plus the Mach error,

$$vg_{ITP} - vg_{Ref} \leq 576.6(0.04 - \eta_{vm\,ITP} + \eta_{vm\,Ref}) \quad (3.6)$$

3.2 Second Wind Model

The second model of wind considers a linearly negative wind gradient from the ITP aircraft flight level to the Reference aircraft flight level as shown in Figure 3.2.

Figure 3.2. Decreasing Gradient Model of Wind.



Similar to the increasing gradient model, the closure rate is bounded by the ITP criteria. The initial ground speed of the ITP and Reference aircraft are given by,

$$vg_{ITP}^{ic} = 576.6\, vm_{ITP}^{ic} + W_{ITP} \quad (3.7)$$

$$vg_{Ref}^{ic} = 576.6\, vm_{Ref}^{ic} + W_{Ref} \quad (3.8)$$

The ground speed of the ITP and Reference aircraft through the maneuver are given by equations 3.2 and 3.3. The ITP procedure requires that the Mach speed of both aircraft remains constant during the maneuver. Hence,

$$vm_{ITP} = vm_{ITP}^{ic} \tag{3.9}$$

$$vm_{Ref} = vm_{Ref}^{ic} \tag{3.10}$$

Because the wind magnitude is decreasing through the maneuver,

$$W_{ITP} \geq W \tag{3.11}$$

From equations 3.2, 3.7, 3.9, and 3.11 we can see that,

$$vg_{ITP} \leq vg_{ITP}^{ic} \tag{3.12}$$

From equations 3.3, 3.8, and 3.10,

$$vg_{Ref} = vg_{Ref}^{ic} \tag{3.13}$$

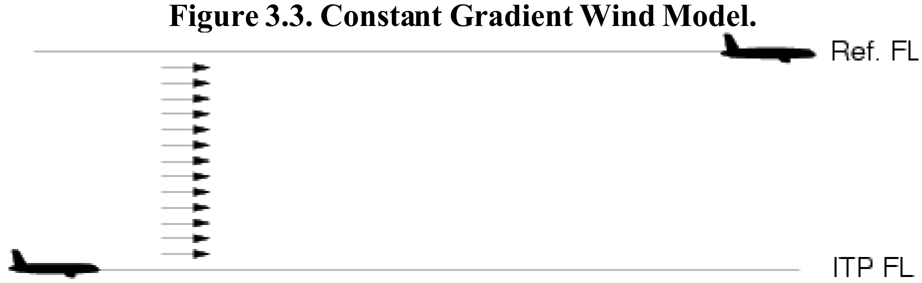
That is, the ground speed of the ITP aircraft through the maneuver is less than or equal to its initial ground speed. The ground speed of the Reference aircraft remains constant. The ITP criteria bounds the ground speed closure rate to 20 knots as shown in Equation 2.8. Therefore, the closure rate everywhere through the maneuver is less than or equal to 20 knots plus the ground speed error,

$$vg_{ITP} - vg_{Ref} \leq 20 - \eta_{vg\,ITP} + \eta_{vg\,Ref} \text{ knots} \tag{3.14}$$

3.3 Third Wind Model

The third model is a zero gradient as shown in Figure 3.3.

Figure 3.3. Constant Gradient Wind Model.



For this model of wind,

$$W_{ITP} = W_{Ref} = W \tag{3.15}$$

From equations 3.2, 3.7, 3.9, and 3.15,

$$vg_{ITP} = vg_{ITP}^{ic} \tag{3.16}$$

Combining equations 2.8, 3.13, and 3.16,

$$vg_{ITP} - vg_{Ref} \leq 20 - \eta_{vg\,ITP} + \eta_{vg\,Ref} \tag{3.17}$$

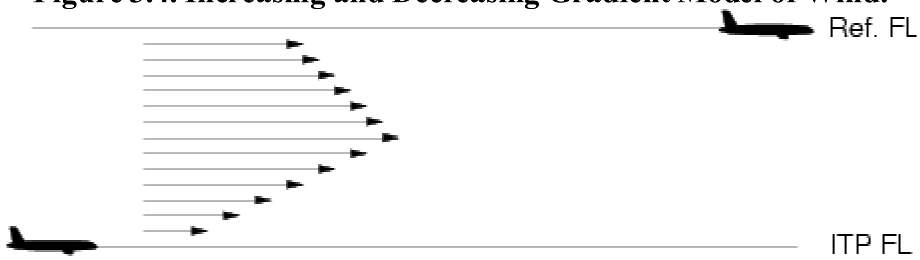
Therefore, the closure rate through the maneuver is less than or equal to 20 knots plus the ground speed error. A similar argument could be made for the Mach speed difference for the constant wind gradient. The closure rate is bounded by the Mach criterion,

$$vg_{ITP} - vg_{Ref} \leq 576.6(0.04 - \eta_{vm_{ITP}} + \eta_{vm_{Ref}}) \tag{3.18}$$

3.4 Fourth Wind Model

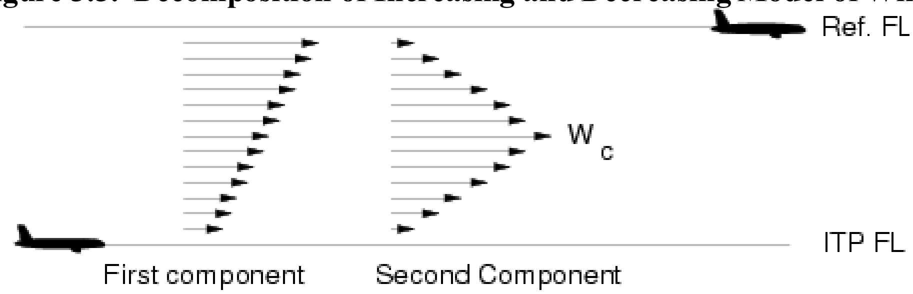
The fourth model of wind considers a positive and then negative wind gradient from the ITP to the Reference aircraft flight levels. This is shown in Figure 3.4. This wind model is the worst case scenario.

Figure 3.4. Increasing and Decreasing Gradient Model of Wind.



This wind model can be analyzed by decomposing the wind gradient into two components: 1. a linear positive, negative or zero gradient; 2. a positive and then negative gradient starting and ending with zero magnitude. This decomposition is shown in Figure 3.5.

Figure 3.5. Decomposition of Increasing and Decreasing Model of Wind.



For the first component of the wind gradient shown in Figure 3.5, we established that the closure rate is determined by the ground speed and Mach criteria. To this closure rate, we add the second component. The average closure rate produced by the second component is $\frac{W_C}{2}$, where W_C is the magnitude of the wind at the middle level. Note that this is not the maximum magnitude of the wind. The maximum magnitude is the sum of the first and second components. W_C represents the rate of change of wind speed over vertical distance. The variable W_C is modeled as a random variable and it is based on wind observations on the North Atlantic and other weather data [4].

For the first component of wind, the worst closure rate when the ITP aircraft is at its own level is,

$$vg_{ITP} - vg_{Ref} = 20 - \eta_{vg\,ITP} + \eta_{vg\,Ref} \tag{3.19}$$

The worst closure rate when the ITP and Reference are at the same altitude is,

$$vg_{ITP} - vg_{Ref} = 576.6\left(0.04 - \eta_{vm_{ITP}} + \eta_{vm_{Ref}}\right) \tag{3.20}$$

The worst average closure rate is the average of the worst closure rates,

$$vg_{ITP} - vg_{Ref} = \frac{576.6(0.04 - \eta_{vm\,ITP} + \eta_{vm\,Ref}) + 20 - \eta_{vg\,ITP} + \eta_{vg\,Ref}}{2} \tag{3.21}$$

The second component of wind is added to Equation 3.21,

$$vg_{ITP} - vg_{Ref} = \frac{576.6(0.04 - \eta_{vm\,ITP} + \eta_{vm\,Ref}) + 20 - \eta_{vg\,ITP} + \eta_{vg\,Ref}}{2} + \frac{W_C}{2} \tag{3.22}$$

Equation 3.22 represents the worst case scenario of the aircraft closure rates considering ground speed and Mach initiation criteria, ground speed errors, Mach errors, and an positve and then negative wind gradient.

4 Distance Error and Wind Model

The distance between the aircraft as a function of time is calculated using the error models and the forth wind model of Section 3. The distance between the aircraft is given by the difference between their initial position plus their velocity multiplied by time.

$$x_{Ref}(t) - x_{ITP}(t) = x_{Ref} + V_{ref}t - (x_{ITP} + V_{ITP}t) \tag{4.1}$$

taking into account position errors, velocity errors, latency error, and wind we combine equations 2.3, 2.15, 3.22, and 4.1,

$$\begin{aligned} x_{Ref}(t) - x_{ITP}(t) = {} & \hat{x}_{Ref} - \hat{x}_{ITP} - \eta_{pRef} + \eta_{pITP} - \\ & \left(\frac{576.6(0.04 - \eta_{vm\,ITP} + \eta_{vm\,Ref}) + 20 - \eta_{vgITP} + \eta_{vg\,Ref} + W_C}{2} \right) t - \\ & \eta_{latency} 0.85(576.6) \end{aligned} \tag{4.2}$$

An ITP maneuver can be performed when the ITP and Reference aircraft are at 1000, 2000, and 3000 feet apart vertically. The minimum climb rate to perform an ITP procedure is 300 feet/minute. The nominal time required to climb from the ITP to the Reference aircraft level is,

$$T = 0.055556 \text{ hours} \qquad \text{for 1000 feet} \tag{4.3}$$

$$T = 0.11111 \text{ hours} \qquad \text{for 2000 feet} \tag{4.4}$$

$$T = 0.16667 \text{ hours} \qquad \text{for 3000 feet} \tag{4.5}$$

The total time to climb/descent to the same flight level is the nominal time plus the time when considering the altitude error.

The true longitudinal distance follows from equation 4.2 and is represented by the random variable Ψ,

$$\Psi = \hat{x}_{Ref} - \hat{x}_{ITP} - \eta_{pRef} + \eta_{pITP} -$$

$$\left(\frac{576.6(0.04-\eta_{vm\,ITP}+\eta_{vm\,Ref})+20-\eta_{vg\,ITP}+\eta_{vg\,Ref}+W_C}{2}\right)(T+t_a)-$$

$$\eta_{latency}0.85(576.6) \qquad (4.6)$$

with mean value,

$$E\{\Psi\} = \hat{x}_{Ref}-\hat{x}_{ITP}-21.53(T+t_a)-\eta_{latency}490.11 \qquad (4.7)$$

and variance,

$$\sigma^2\{\Psi\} = \sigma^2_{p\,Ref}+\sigma^2_{p\,ITP}+\frac{\left[576.6^2(\sigma^2_{vm\,ITP}+\sigma^2_{vm\,Ref})+\sigma^2_{vg\,ITP}+\sigma^2_{vg\,Ref}+\sigma^2_{W_C}\right]}{2^2}(T+t_a)^2 \qquad (4.8)$$

where $\sigma_{p\,Ref}$, $\sigma_{p\,ITP}$, $\sigma_{vm\,ITP}$, $\sigma_{vm\,Ref}$, $\sigma_{vg\,ITP}$, $\sigma_{vg\,Ref}$, and σ_{W_C} are the standard deviations of the position, Mach, and ground speed errors and wind speed W_C.

5 Types of Error Distributions

5.1 Introduction

The errors considered in the collision risk analysis can be modeled depending on the characteristics of the error. Two distributions are used to model the position and velocity errors and the wind speed: A double exponential distribution also known as the First Laplace; A Normal distribution also known as Gaussian.

Table 5.1. Types of probability distributions used in calculations.

Type of error or wind		***Type of probability distribution***	
Description	**Symbol and Units**	**1st choice**	**2nd choice**
Position errors	$\eta_{p\,ITP}$ $\eta_{p\,Ref}$ NM	Double exponential	Normal
Ground speed errors	$\eta_{vg\,ITP}$ $\eta_{vg\,Ref}$ knots	Double exponential	Normal
Mach error	$\eta_{vm\,ITP}$ $\eta_{vm\,Ref}$ Mach	Double exponential	Normal
Altitude error	$\eta_{alt\,ITP}$ $\eta_{alt\,Ref}$ feet	Constant	Constant
Latency error	$\eta_{latency}$ seconds	Constant	Constant
Wind	W_C knots	Double exponential	Normal

5.2 Values used for distributions

The values used for the distributions for position and ground speed errors are based on the

95-quantile accuracy numbers. The standard deviation for the Normal distribution is the 95-quantile divided by 1.96. The standard deviation for the Double exponential distribution is the 95-quantile divided by 2.118.

The position error is based on the required accuracy for the ITP initiation criterion. For both the ITP and Reference aircraft, the position error 95 percentile is 0.3 NM. The ground speed error is also determined by the required initiation criterion and has a 95 percentile error of 6.75 meters per second (13.12 knots). The Mach error was provided by avionics equipment manufacturers. The wind non-linearity standard deviation is based on reference [4]. The altitude error is based on RVSM minimum certification standards. A summary of the RVSM Airworthiness Approval – Altimeter System Error is shown in Table 5.2

Table 5.2. Airworthiness Approval – Altimeter System Error for RVSM

Requirements: Group Airplanes		
Type Certificate issued before 1st January 1997	Normal Operating Conditions (Basic Flight Envelope)	Full Operating Capabilities (Full Flight Envelope)
Mean Error	80 feet	120 feet
Mean Error and Three Standard Deviations	200 feet	245 feet
Type Certificate issued after 1st January 1997	Normal Operating Conditions (Basic Flight Envelope)	Full Operating Capabilities (Full Flight Envelope)
Mean Error	80 feet	80 feet
Mean Error and Three Standard Deviations	200 feet	200 feet
Requirements: Non-Group Airplanes		
	Normal Operating Conditions (Basic Flight Envelope)	Full Operating Capabilities (Full Flight Envelope)
Absolute Error	160 feet	200 feet

For type certificates before January 1997, the maximum standard deviation is 81.66 feet when the mean error is zero and 41.66 feet when the mean error is 120 feet. For type certificates on or after January 1997, the maximum standard deviation is 66.66 feet when the mean is zero and 40 feet when the mean is 80 feet. For non-group aircraft, the absolute maximum error is 200 feet. We take 400 feet as the constant error for altitude error,

$$\eta_{alt\ ITP} + \eta_{alt\ Ref} = 400 \text{ feet} \tag{5.1}$$

The latency error is assumed to always contribute to the reduction in distance between the aircraft. The latency error is taken to be constant to simplify the collision analysis and it is based on the Minimum Aviation System Performance Standards, MASPS for ADS-B [6,7].

The MASPS specify:

"For NUCp less than 8, ADS-B latency of the reported information shall be less than 1.2 seconds with 95 percent confidence."

(NUC – Navigation Uncertainty Category)

When an aircraft transmits NAC and NIC, the requirements is for a latency of 1.2 seconds at 95 percent when NAC is less than10 and NIC is less than 9.

(NAC – Navigation Accuracy Category)

(NIC – Navigation Integrity Category)

A NUCp of 5 or greater has been defined as a requirement for the ITP application. When an aircraft is transmitting NAC and NIC, the requirement is for NAC and NIC of 6 or greater. Hence, the latency shall be 1.2 seconds with a 95 percent confidence.

For the results shown in this report, the latency error is assumed to be a constant 4.575 seconds. The constant latency represents a 1.0×10^{-5} bound on the latency error assuming an exponential distribution with 1.2 seconds (or less) latency 95 percent of the time.

$$\eta_{latency} = 4.575 \text{ seconds} = 0.0012708333 \text{ hours} \tag{5.2}$$

Table 5.3 shows the standard deviation and scale parameter values for the position, ground speed, and Mach errors and wind for both distributions.

Table 5.3. Standard deviations and scale parameter of error distributions.

Error Type or wind	σ ***Normal***	σ ***Double Exponential***	λ ***Double Exponential***
Position	0.153061 NM	0.141643 NM	0.100157 NM
Ground speed	3.44348 meters/sec. 6.69359 knots	3.18660 meters/sec. 6.19426 knots	2.25325 meters/sec. 4.38000 knots
Mach error	0.0010 Mach	0.0009254 Mach	0.0006544 Mach
Wind	1.5445 knots	1.4293 knots	1.0106 knots

The value for position error correspond to an ADS-B transmission with a NUCp = 5, or NACp = 6, NIC = 6, and SIL = 2. The value for ground speed error was selected such that it is the minimum requirement that meets a collision risk of less than 1E-09 for the double exponential distribution and a 3000 feet vertical distance between the aircraft (a 4000 feet climb.) The values on the table for ground speed error correspond to an error of 13.11944 knots at 95 percent confidence.

6 Probability of Longitudinal Overlap

Previous SASP safety analyses of overlap probability and collision risk for similar distance based separation procedures have not used wind as a contributing factor in the calculations. In the analysis reported in this paper, we consider the surveillance error and add a wind model to the longitudinal overlap probability.

6.1 Probability Density Function, 1st Choice of Distributions, Double Exponential

The probability density function of the random variable Ψ, as defined by equation 4.6, is calculated using the double exponential distributions. The ground speed errors, Mach errors, and wind are normalized to nautical miles,

$$\varepsilon_{vg} = \eta_{vg} \frac{(T+t_a)}{2} \text{ NM} \tag{6.1}$$

$$\varepsilon_{vm} = \eta_{vm} 576.6 \frac{(T+t_a)}{2} \text{ NM} \tag{6.2}$$

$$\varepsilon_W = W_C \frac{(T+t_a)}{2} \text{ NM} \tag{6.3}$$

We can rewrite equation 4.6, using the normalized random variables, into the sum of a constant plus 7 random variables,

$$\Psi = C + \eta_{p\,ITP} + \eta_{p\,Ref} + \varepsilon_{vg\,ITP} + \varepsilon_{vg\,Ref} + \varepsilon_{vm\,ITP} + \varepsilon_{vm\,Ref} + \varepsilon_W \tag{6.4}$$

where,

$$C = \hat{x}_{Ref} - \hat{x}_{ITP} - \frac{(576.6(0.04)+20)}{2}(T+t_a) - \eta_{latency} 0.85(576.6) \tag{6.5}$$

In the following paragraphs, the subscripts "*ITP*" and "*Ref*" are replace by "1" and "2", respectively, to make the equations more compact.

The probability density function of the sum of the position error η_{p1} and normalized ground speed error ε_{vg1} random variables for the ITP aircraft is given by

$$f_{p1+vg1}(x) = \frac{1}{2}\left[\frac{\lambda_{p1} e^{-|x|/\lambda_{p1}} - \lambda_{vg1} e^{-|x|/\lambda_{vg1}}}{\lambda_{p1}^2 - \lambda_{vg1}^2}\right] \tag{6.6}$$

where $\lambda_{p1} \neq \lambda_{\upsilon 1}$.

A similar density function is given for the Reference aircraft position and ground speed errors,

$$f_{p2+vg2}(x) \;=\; \frac{1}{2}\left[\frac{\lambda_{p2} e^{-|x|/\lambda_{p2}} - \lambda_{vg2} e^{-|x|/\lambda_{vg2}}}{\lambda_{p2}^2 - \lambda_{vg2}^2}\right] \tag{6.7}$$

where $\lambda_{p2} \neq \lambda_{vg2}$.

The probability density function of the sum of the position and ground speed error random variables for both aircraft is the convolution of their probability density functions, equations 6.6 and 6.7,

$$f_{p1+vg1+p2+vg2}(x) \;=\; \int_{-\infty}^{\infty} f_{p1+vg1}(x-s)\, f_{p2+vg2}(s)\, ds \tag{6.8}$$

$$f_{p1+vg1+p2+vg2}(x) \;=\; \int_{-\infty}^{\infty} \frac{1}{2}\left[\frac{\lambda_{p1} e^{-|x-s|/\lambda_{p1}} - \lambda_{vg1} e^{-|x-s|/\lambda_{vg1}}}{\lambda_{p1}^2 - \lambda_{vg1}^2}\right] \frac{1}{2}\left[\frac{\lambda_{p2} e^{-|x|/\lambda_{p2}} - \lambda_{vg2} e^{-|x|/\lambda_{vg2}}}{\lambda_{p2}^2 - \lambda_{vg2}^2}\right] ds \tag{6.9}$$

Because we assume that the position and velocity errors have the same distribution for the ITP and Reference aircraft, we let

$$\lambda_p \;=\; \lambda_{p1} \;=\; \lambda_{p2} \tag{6.10}$$

$$\lambda_{vg} \;=\; \lambda_{vg1} \;=\; \lambda_{vg2} \tag{6.11}$$

in equation 6.9. This results in a probability density function for both aircraft position and velocity errors given by,

$$f_{p+vg}(x) \;=\; \frac{1}{4(\lambda_p^2 - \lambda_{vg}^2)^2}\left[\left(\lambda_p^2|x| + \lambda_p^3 - \frac{4\lambda_{vg}^2\lambda_p^3}{\lambda_p^2 - \lambda_{vg}^2}\right) e^{-|x|/\lambda_p} + \left(\lambda_{vg}^2|x| + \lambda_{vg}^3 + \frac{4\lambda_{vg}^3\lambda_p^2}{\lambda_p^2 - \lambda_{vg}^2}\right) e^{-|x|/\lambda_{vg}}\right] \tag{6.12}$$

The probability density function of the Mach errors is give by,

$$f_{vm1+vm2}(x) \;=\; \frac{1}{4\lambda_{vm}^2}\left[|x| e^{-|x|/\lambda_{vm}} + \lambda_{vm} e^{-|x|/\lambda_{vm}}\right] \tag{6.13}$$

where,

$$\lambda_{vm} = \lambda_{vm1} = \lambda_{vm2} \qquad (6.14)$$

The probability density function of the Mach errors and wind is the convolution of equation 6.13 with the wind double exponential distribution and is given by,

$$f_{vm+W}(x) = \frac{1}{4(\lambda_{vm}\lambda_W^2 - \lambda_{vm}^3)}\left(F e^{-|x|/\lambda_W} + G e^{-|x|/\lambda_{vm}} - |x|\lambda_{vm} e^{-|x|/\lambda_{vm}}\right) \qquad (6.15)$$

where,

$$F = \frac{2\lambda_W^3\lambda_{vm}}{\lambda_W^2 - \lambda_{vm}^2} \qquad (6.16)$$

$$G = -\left(\lambda_{vm}^2 + \frac{2\lambda_W^2\lambda_{vm}^2}{\lambda_W^2 - \lambda_{vm}^2}\right) \qquad (6.17)$$

The probability density function of all 7 random variables is the convolution of equation 6.12 with equation 6.15,

$$\begin{aligned}
f_7(x) = {} & \frac{1}{D}\left[F\left(\frac{2|x|\lambda_W\lambda_p^4}{\lambda_p^2-\lambda_W^2} - \frac{4\lambda_W^3\lambda_p^5}{(\lambda_p^2-\lambda_W^2)^2} + \frac{A\,2\lambda_W\lambda_p^2}{\lambda_p^2-\lambda_W^2}\right)\right]e^{-|x|/\lambda_p} \\
& + \frac{1}{D}\left[G\left(\frac{2|x|\lambda_{vm}\lambda_p^4}{\lambda_p^2-\lambda_{vm}^2} - \frac{4\lambda_{vm}^3\lambda_p^5}{(\lambda_p^2-\lambda_{vm}^2)^2} + \frac{A\,2\lambda_{vm}\lambda_p^2}{\lambda_p^2-\lambda_{vm}^2}\right)\right]e^{-|x|/\lambda_p} \\
& + \frac{1}{D}\left[\left(\frac{-2|x|\lambda_{vm}^3\lambda_p^6 - 2|x|\lambda_{vm}^5\lambda_p^4}{(\lambda_p^2-\lambda_{vm}^2)^2} + \frac{4\lambda_{vm}^7\lambda_p^5 + 12\lambda_{vm}^5\lambda_p^7}{(\lambda_p^2-\lambda_{vm}^2)^3} - \frac{2\lambda_p^2\lambda_{vm}^5 A + 2\lambda_p^4\lambda_{vm}^3 A}{(\lambda_{vm}^2-\lambda_p^2)^2}\right)\right]e^{-|x|/\lambda_p} \\
& + \frac{1}{D}\left[F\left(\frac{2|x|\lambda_W\lambda_{vg}^4}{\lambda_{vg}^2-\lambda_W^2} - \frac{4\lambda_W^3\lambda_{vg}^5}{(\lambda_{vg}^2-\lambda_W^2)^2} + \frac{A\,2\lambda_W\lambda_{vg}^2}{\lambda_{vg}^2-\lambda_W^2}\right)\right]e^{-|x|/\lambda_{vg}} \\
& + \frac{1}{D}\left[G\left(\frac{2|x|\lambda_{vm}\lambda_{vg}^4}{\lambda_{vg}^2-\lambda_{vm}^2} - \frac{4\lambda_{vm}^3\lambda_{vg}^5}{(\lambda_{vg}^2-\lambda_{vm}^2)^2} + \frac{A\,2\lambda_{vm}\lambda_{vg}^2}{\lambda_{vg}^2-\lambda_{vm}^2}\right)\right]e^{-|x|/\lambda_{vg}} \\
& + \frac{1}{D}\left[\left(\frac{-2|x|\lambda_{vm}^3\lambda_{vg}^6 - 2|x|\lambda_{vm}^5\lambda_{vg}^4}{(\lambda_{vg}^2-\lambda_{vm}^2)^2} + \frac{4\lambda_{vm}^7\lambda_{vg}^5 + 12\lambda_{vm}^5\lambda_{vg}^7}{(\lambda_{vg}^2-\lambda_{vm}^2)^3} - \frac{2\lambda_{vg}^2\lambda_{vm}^5 A + 2\lambda_{vg}^4\lambda_{vm}^3 A}{(\lambda_{vm}^2-\lambda_{vg}^2)^2}\right)\right]e^{-|x|/\lambda_{vg}}
\end{aligned}$$

$$+\frac{1}{D}\left[G\left(\frac{2\lambda_{vm}^2\lambda_p^6+\lambda_{vm}^4\lambda_p^4}{(\lambda_p^2-\lambda_{vm}^2)^2}-\frac{A2\lambda_{vm}^2\lambda_p}{\lambda_p^2-\lambda_{vm}^2}-\frac{B2\lambda_{vm}^2\lambda_{vg}}{\lambda_{vg}^2-\lambda_{vm}^2}\right)+\left(\frac{-2|x|\lambda_{vm}^3\lambda_p^6-2|x|\lambda_{vm}^5\lambda_p^4}{(\lambda_p^2-\lambda_{vm}^2)^2}\right)\right]e^{-|x|/\lambda_{vm}}$$

$$+\frac{1}{D}\left[\left(\frac{-(4\lambda_{vm}^4\lambda_p^8+12\lambda_{vm}^6\lambda_p^6)}{(\lambda_p^2-\lambda_{vm}^2)^3}-\frac{2|x|\lambda_{vm}^3\lambda_{vg}^6+2|x|\lambda_{vm}^5\lambda_{vg}^4}{(\lambda_{vg}^2-\lambda_{vm}^2)^2}-\frac{(4\lambda_{vm}^4\lambda_{vg}^8+12\lambda_{vm}^6)}{(\lambda_{vg}^2-\lambda_{vm}^2)^3}\right)\right]e^{-|x|/\lambda_{vm}}$$

$$+\frac{1}{D}\left[\left(\frac{-2|x|\lambda_p\lambda_{vm}^3A}{\lambda_{vm}^2-\lambda_p^2}+\frac{4\lambda_p^3\lambda_{vm}^4A}{(\lambda_{vm}^2-\lambda_p^2)^2}-\frac{2|x|\lambda_{vg}\lambda_{vm}^3B}{\lambda_{vm}^2-\lambda_{vg}^2}+\frac{4\lambda_{vg}^3\lambda_{vm}^4B}{(\lambda_{vm}^2-\lambda_{vg}^2)^2}\right)\right]e^{-|x|/\lambda_{vm}}$$

$$+\frac{1}{D}\left[F\left(\frac{2\lambda_W^2\lambda_p^6+2\lambda_W^4\lambda_p^4}{(\lambda_p^2-\lambda_W^2)^2}+\frac{2\lambda_W^2\lambda_{vg}^6+2\lambda_W^4\lambda_{vg}^4}{(\lambda_{vg}^2-\lambda_W^2)^2}-\frac{A2\lambda_W^2\lambda_p}{\lambda_p^2-\lambda_W^2}-\frac{B2\lambda_W^2\lambda_{vg}}{\lambda_{vg}^2-\lambda_W^2}\right)\right]e^{-|x|/\lambda_W} \qquad (6.18)$$

where F and G are defined by equations 6.16 and 6.17, respectively, and

$$D = 16(\lambda_{vm}\lambda_W^2-\lambda_{vm}^3)(\lambda_p^2-\lambda_v^2)^2 \qquad (6.19)$$

$$A = \lambda_p^3-\frac{4\lambda_{vg}^2\lambda_p^3}{\lambda_p^2-\lambda_{vg}^2} \qquad (6.20)$$

$$B = \lambda_{vg}^3+\frac{4\lambda_{vg}^3\lambda_p^2}{\lambda_p^2-\lambda_{vg}^2} \qquad (6.21)$$

The first term of equation 6.4 is a constant term representing the measured distance and the reduction in distance due to ground speed, Mach differences and latency error. The constant term of the random variable is added to the probability density function by substituting the term $x-C$ everywhere the term x appears in Equation 6.18.

6.1.1 Probability of Longitudinal Overlap at Co-altitude; Double Exponential

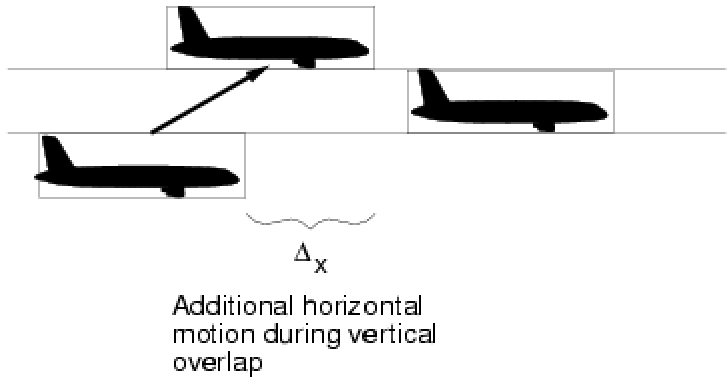
The probability of longitudinal overlap is the probability that the ITP aircraft, represented by a point in space, is within the longitudinal space of the Reference aircraft, represented by 2 times the length of the aircraft, at the time that both aircraft are at the same flight level. This is represented by

$$x(t) \in [-\lambda_x, \lambda_x] \qquad (6.22)$$

where λ_x is the length of the aircraft. The value of λ_x is taken as the longest commercial aircraft length (Airbus 380) of 0.0392818 NM (72.75 m, 238.67 feet).

The In-Trail Procedure is a climb or descent maneuver and the ITP aircraft might have a closing speed on the Reference aircraft. Therefore, the longitudinal overlap interval is increased by the relative ground speed times the time it takes the two aircraft to pass vertically. This is illustrated in Figure 6.1.

Figure 6.1. Horizontal Distance Gained by ITP Aircraft during Vertical Overlap.



The time t_V that takes the ITP aircraft to vertically pass the Reference aircraft is given by 2 times the hight of the aircraft λ_z divided by the vertical speed. The hight of the aircraft is the hight of the largest commercial aircraft (Airbus 380), 24.08 m (79 feet).

$$t_V = \frac{2\lambda_z}{vs} = \frac{2\times 79\ feet}{300\ ft/min} = 0.00877778\ \text{hours} \tag{6.23}$$

The additional horizontal distance Δ_x is the time t_V times the maximum horizontal relative speed, 0.04 Mach (23.064 knots at 35K feet and ISA).

$$\Delta_x = 23.064\times 0.00877778 = 0.202451\ \text{NM} \tag{6.24}$$

The probability of longitudinal overlap during vertical overlap is then given by the integration of the probability density function of the longitudinal distance, Equation 6.18, over the interval $[-\lambda_x,\ \lambda_x+\Delta_x]$.

$$P_x[-\lambda_x < x(t) < \lambda_x+\Delta_x] = \int_{-\lambda_x}^{\lambda_x+\Delta_x} f_7(x)dx \tag{6.25}$$

The probability of longitudinal overlap for 1000, 2000, and 3000 feet vertical distance is given in Table 6.1.

Table 6.1. Probability of Longitudinal Overlap, Double Exponential Distributions.

Altitude Difference, feet	$P_x[-\lambda_x < x(t) < \lambda_x + \Delta_x]$
1000	7.82863×10^{-30}
2000	1.62178×10^{-15}
3000	9.67251×10^{-10}

6.2 Probability Density Function, 2nd Choice of Distributions, Normal

The probability density function of the random variable Ψ , as defined by equations 4.6 and 6.4, is calculated using the Normal (Gaussian) distributions. The sum of random variables with Normal distributions result in a random variable with a Normal distribution. From equation 4.8, the variance of the random variable Ψ is the sum of the individual variances and the standard deviation is the positive square root of the variance.

For the Normal distributions, the probability density function is given by,

$$f_{p+vg+vm+W+C}(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-C)^2/2\sigma^2} \tag{6.17}$$

where σ is obtained from Equation 4.8 and C is defined by Equation 6.5. The values used for σ and C are shown in Table 6.2.

Table 6.2. Standard Deviation and Mean for Normal Distribution.

Altitude Difference, feet	*Standard Deviation,* σ	*Mean, C*
1000	0.432420 NM	12.702441 NM
2000	0.677253 NM	11.506218 NM
3000	0.934531 NM	10.309996 NM

6.2.1 Probability of Longitudinal Overlap at Co-altitude; Normal

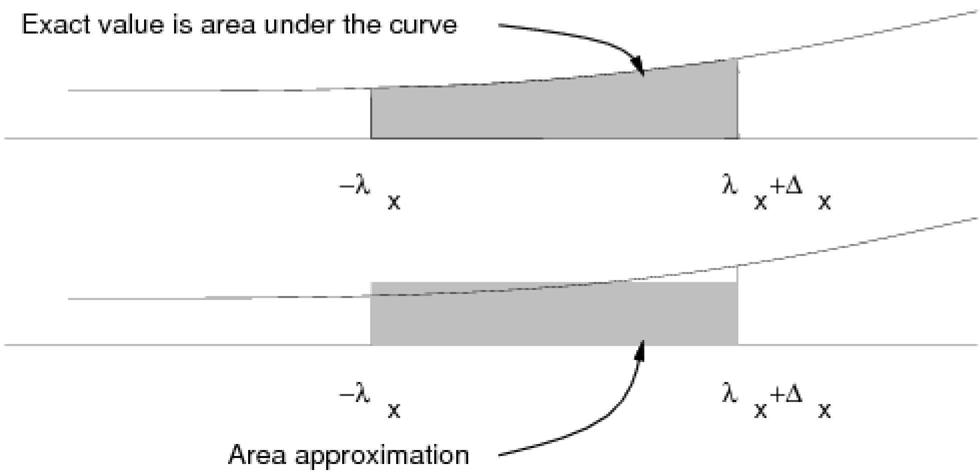
The probability of longitudinal overlap during vertical overlap is given by the integration of the probability density function of the longitudinal distance, Equation 6.17, over the interval $[-\lambda_x,\ \lambda_x+\Delta_x]$.

$$P_x[-\lambda_x < x(t) < \lambda_x + \Delta_x] = \int_{-\lambda_x}^{\lambda_x+\Delta_x} f_{p+vg+vm+W+C}(x)\,dx \tag{6.18}$$

Because the distribution is normal, in most cases the integral of Equation 6.18 can be evaluated using the error function. However, due to the extremely small numbers which are computed when evaluating an integral which is more than 10 standard deviations from the mean, the error function, as implemented in the Java programing language, computes to a value of zero.

An approximation is used to evaluate the integral of Equation 6.18. The probability density function, Equation 6.17, is evaluated at the upper and lower limits of the integral. The values of the probability density function at the limits are added and divided by 2 to obtain the average. The average is multiplied by the difference of the limits to obtain an approximation of the area under the curve as shown in Figure 6.2.

Figure 6.2. Integral Calculation by Approximation of Area under the Curve.



The probability of longitudinal overlap for 1000, 2000, and 3000 feet vertical distance is given in Table 6.3.

Table 6.3. Probability of Longitudinal Overlap, Normal (Gaussian) Distributions.

Altitude Difference, feet	$P_x[-\lambda_x < x(t) < \lambda_x + \Delta_x]$
1000	6.30217×10^{-182}
2000	7.01226×10^{-62}
3000	6.98290×10^{-27}

7 References

[1] Carreño, V. and Muñoz, C.; Safety and Performance Analysis of the Non-Radar Oceanic/Remote Airspace In-Trail Procedure, NASA-TM-2007-214856, March 2007.

[2] ICAO Doc 4444 – PANS-ATM

[3] Minimum Operational Performance Standards for 1090 MHz Automatic Dependent Surveillance – Broadcast (ADS-B), RTCA/DO-260, September 2000.

[4] Carreño, V. and Chartrand, R.; Wind Characterization for the Assessment of Collision Risk During Flight Level Changes, November 2007.

[5] Requirement Focus Group; Operational Services and Environment Definition, In-Trail Procedure, May, 2007.

[6] RTCA DO-242, Minimum Aviation System Performance Standards for Automatic Dependent Surveillance Broadcast (ADS-B), February, 1998.

[7] RTCA DO-242A, Minimum Aviation System Performance Standards for Automatic Dependent Surveillance Broadcast (ADS-B), June, 2002.

Appendix A

The objective of this appendix is to illustrate, through examples, the interrelation between the ground speed error, the Mach error and the wind.

The examples below refer to the variables depicted in Figure A.1. The variable are as follow:

W_{ITP} wind along the track at the flight level of the ITP aircraft

W_{Ref} wind along the track at the flight level of the Reference aircraft

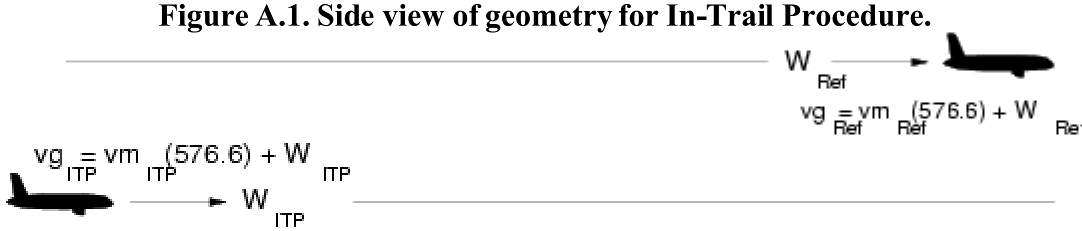
vm_{ITP} true Mach air speed of the ITP aircraft

vm_{Ref} true Mach air speed of the Reference aircraft

vg_{ITP} true ground speed along the track of the ITP aircraft

vg_{Ref} true ground speed along the track of the Reference aircraft

Figure A.1. Side view of geometry for In-Trail Procedure.



Example 1:

W_{ITP} = 100 knots; W_{Ref} = 104 knots; vm_{ITP} = 0.84 Mach; vm_{Ref} = 0.80 Mach;

These conditions result in the following (at FL350 with International Standard Atmosphere):

vg_{ITP} = 584.3 knots; vg_{Ref} = 565.3 knots; Δ ground speed = 19.0 knots; Δ Mach = 0.04

Hence, the ground speed and Mach criteria are met.

Example 2:

Same conditions as Example 1, but with a large Mach error on the ITP aircraft. The ITP has a commanded Mach of 0.84 but its true Mach is 0.87.

These conditions results in the following:

vg_{ITP} = 601.6 knots; vg_{Ref} = 565.3 knots; Δ ground speed = 36.3 knots; Δ Mach = 0.04 (measured)

In this example, the ground speed difference exceeds the criterion. The ITP procedure should not be requested or executed. Although the error is in the Mach measurement, the error is bounded by the ground speed measurement.

Example 3:

W_{ITP} = 100 knots; W_{Ref} = 121 knots; vm_{ITP} = 0.87 Mach; vm_{Ref} = 0.80 Mach;

The ITP aircraft has a large Mach speed error and has an indicated (measured) Mach number of 0.84.

These conditions result in the following:

vg_{ITP} = 601.6 kts; vg_{Ref} = 582.3 kts; Δ ground speed = 19.3 knots; Δ Mach = 0.04

The ITP criteria for ground and Mach speeds are met due to the error. The closure rate on the aircraft is 19.3 knots at the beginning of the maneuver and increases to 40.4 knots at co-altitude. Assuming a 3000 feet vertical distance, 300 feet/minute climb/descent rate, a linear wind gradient, and a 15 NM initial distance, the error in this example will reduce the distance between the aircraft to 10.02 NM at co-altitude.

Remarks

A large error in Mach speed will result in conditions that will likely affect the ground speed criterion, unless the wind conditions are such that hide the error. Conversely, a large ground speed error will likely be bounded by the Mach criterion.

Also, the difference in Mach speed of commercial aircraft is limited to approximately 0.12 Mach. This is based on flight plans data filed for the North Atlantic on the 7th March 2005, reference [A1]. This data shows that the majority of aircraft have a Mach difference of 0.06 or less.

Hence, the error distribution of the Mach speed is affected both by the ground speed criterion and by the physical characteristics of the aircraft which will be performing the ITP procedure. The ground speed error distribution and probability density function are in turn affected by the Mach criterion.

References

[A1] Carreño, Victor; Percentage of Cases Meeting the ITP Initiation Criteria and Probability of Loosing Separation during an ITP Maneuver, February 2007, unpublished. Available at http://shemesh.larc.nasa.gov/people/vac/papers.html.